B. Math. I

Mid - term Examination
Algebra - I

- **All questions are compulsory.**
- **All questions carry equal marks.**

(1) Let G be a finite group such that for every prime p and natural number r, if p^r divides it's order then there exists an element of order p^r in G. Prove that G is abelian if and only if G is cyclic.

(2) State and prove Lagrange's Theorem.

(3) (a) Determine all elements of order 8 in $Z/8888888Z$

(b) State and prove generalization of (a).

(4) Let G be a finite cyclic group of order n. Find $\text{Aut}(G)$.

(5) (a) Let Φ from G to G' be a group homomorphism with kernel K. Let H be another subgroup of G containing K. Show that $(\Phi)^{-1}\Phi(H) = H$.

(b) Let Let Φ from G to G' be a surjective group homomorphism with kernel K. Show that the set of subgroups H' of G' is in bijective correspondence with the set of subgroups H of G which contains K, the correspondence being defined by the map H goes to $\Phi(H)$ and $(\Phi)^{-1}(H')$ goes to H'. Moreover normal subgroups of G correspond to normal subgroups of G'.

(6) Compute the group $(Z \times Z)/ < (1,1) >$ by visualizing $Z \times Z$ as the points in the plane with both co-ordinates integers.

(7) Let G be a group of matrices of the form $\begin{pmatrix} x & y \\ 0 & 1 \end{pmatrix}$ and $H = \begin{pmatrix} x & 0 \\ 0 & 1 \end{pmatrix} \in GL_2(R)$. An element of G can be represented by a point in the (x, y) plane. Draw the partition of the plane in to right cosets and left cosets of H.

Is H a normal subgroup of G?